

Mathematics III

Frameworks

Student Edition

Unit 2

Graphs of Polynomial Functions

2nd Edition

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Georgia Department of Education

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Mathematics III – Unit 2

Graphs of Polynomial Functions

Student Edition

INTRODUCTION:

In this unit, students will become familiar with characteristics and graphs of higher order polynomial functions. Students will review characteristics of functions, e.g. domain, range, zeroes, symmetry, that they have previously learned when studying linear, quadratic and cubic functions in Math I and II. In Math III, they will extend their understanding of these concepts to higher order polynomial functions as well as learn new characteristics of these functions. The concepts of multiplicity of real zeroes, relative and absolute extrema, and end behavior will be introduced in this unit. This unit emphasizes graphs of polynomials functions and will require students to graphically find solutions to polynomial equations and understand the meaning of these solutions in context, but does not require students to solve polynomial equations analytically. Students are encouraged to thoroughly understand the behavior of polynomial functions and their roots in Unit 2 before they are asked to find roots of a polynomial function in Unit 4.

ENDURING UNDERSTANDINGS:

- Recognize that polynomials have distinct graphs and characteristics
- Determine the symmetry of a polynomial
- Plot points in three space
- Find the distance between two points in three space

KEY STANDARDS ADDRESSED:

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

- a. Graph simple polynomial functions as translations of the function $f(x) = ax^n$.
- b. Understand the effects of the following on the graph of a polynomial function: degree, lead coefficient, and multiplicity of real zeros.
- c. Determine whether a polynomial function has symmetry and whether it is even, odd, or neither.
- d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3G3. Students will investigate planes and spheres.

- a. Plot the point (x, y, z) and understand it as a vertex of a rectangular prism.
- b. Apply the distance formula in 3-space.

RELATED STANDARDS ADDRESSED:**MM3A3. Students will solve a variety of equations and inequalities.**

- a. Solve polynomial, exponential, and logarithmic equations analytically, graphically, and using appropriate technology.
- d. Solve a variety of types of equations by appropriate means choosing among mental calculation, pencil and paper, or appropriate technology.

MM3P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

MM3P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

MM3P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

MM3P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

MM3P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

UNIT OVERVIEW

This unit consists of five tasks. The first task launches the unit by refreshing students' understandings of what they previously learned in Math I and II about linear, quadratic and cubic functions. The next task uses the motivation of the first unit to understand how these functions students have become familiar with are all polynomials functions. In the spirit of mathematics as a science of patterns, students will investigate characteristics of polynomial functions by examining patterns and testing conjectures they make according to their observations. Students will extend their understanding of polynomial functions to higher order polynomial functions and will become familiar with sketching graphs of polynomial equations and graphically finding the roots of these equations. The third task allows students to take the role of a program evaluator who is in charge of explaining and reporting trends found in a given data set. The students will plot the data and see that the data takes on the shape of a higher order polynomial function. This task focuses on interpreting and explaining characteristics of the polynomial function in context. The next task introduces a problem situation which requires students to extend their understanding of the distance formula from two dimensions to three dimensions in order to solve the problem. The culminating task for this unit is an exciting challenge that requires students to work with and learn from each other and seeks to encourage students solidify their understandings of what they have learned about characteristics and graphs of polynomial functions throughout this unit.

VOCABULARY and FORMULAS

Polynomial function

A *polynomial function* is defined as a function, $f(x)=$

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-2}x^2 + a_{n-3}x^1 + a_n$, where the coefficients are real numbers.

Degree The *degree of a polynomial* (n) is equal to the greatest exponent of its variable.

End Behavior The value of $f(x)$ as x approaches infinity and negative infinity describes the *end behavior* of a polynomial function.

Zero If $f(x)$ is a polynomial function, then the values of x for which $f(x)=0$ are called the *zeros* of the function. Graphically these are the x intercepts. Numbers that are zeros of a polynomial function are also solutions to polynomial equations.

Root Solutions to polynomial equations are called *roots*.

Multiplicity The *multiplicity of a root* refers to the number of times a root occurs at a given point of a polynomial equation.

Relative minimum A *relative minimum* is a point on the graph where the function is increasing as you move away from the point in the positive and negative direction along the horizontal axis.

Relative maximum A *relative maximum* is a point on the graph where the function is decreasing as you move away from the point in the positive and negative direction along the horizontal axis.

Relative Extrema *Relative extrema* refers to relative minimum and relative maximum points.

LAUNCHING TASK: CANTILEVERS LEARNING TASK:

A cantilever is a beam with support on only one end. Cantilevers are often used in constructing bridges, as pictured below:



http://en.wikipedia.org/wiki/Cantilever_bridge

A more common example of a cantilever may be a diving board: one end is anchored for stability and one is not so that it can extend over the pool.



A



flag pole that is mounted horizontally is also a cantilever:



Can you think of other any other types of cantilevers?

When force is applied to the unsupported end of the cantilever, it will bend. This amount of bending depends on different variables, as you may be able to imagine with the diving board example.

What are some of these variables?

Hypothesize how some of these variables are related to the amount of bending of a cantilever.

In order to model this situation and to determine the relationships between some of the variables, we can make a ruler into a cantilever by using a C-clamp to secure the end of the ruler to the end of a flat table. Then, we can apply force to the end of the ruler in order to make it bend by making it support a cup full of pennies.

Part A: Determining the Relationship between Bending and Force

1. First, we want to determine how the amount of bending changes when different amounts of force are applied. How can you collect data with these variables? How will you measure these variables?
2. Once you have decided how to collect your data, you want to record the amount of bending that takes place for 8 different amounts of force. (Discuss with your groups how to vary your amounts of force to be able to see a change in the amount of bending. Here our “force” is the weight of the pennies.)
 - a. Make a table that lists the amount of force for the eight different amounts of force.
 - b. Draw a scatterplot of these values.
3. By the shape of the plotted points, does this function seem to be linear, quadratic, or neither?
 - a. Using the Linear Regression, Quadratic Regression, Cubic Regression, and Quartic Regression functions on your calculator, how can we determine which model fits the data the best? What does it for a model to “fit the data the best?”
 - b. Which model seems to fit the data most closely? How do you know?
 - c. Sketch this function over your scatterplot.
4. Write the equation for the model that best fits your data from your calculator. What type of function is this?
 - a. Give the domain and range for this function.
 - b. Using this function, predict how much the cantilever will bend for _____
 - c. Using this function, predict how much the cantilever will bend if you had 100,000 pennies.
 - d. Does this value make sense? Imagine what the ruler would look like if it was bending this much.
 - e. Recall the domain and range you specified for this function in (a). Consider the context of the problem. Does the context restrict your domain and/or range? (Hint: At what force do you think the cantilever will break? How much can the ruler bend before it breaks?)

5. Is this function increasing or decreasing? In the context, what does that mean?

Part B: Determining the Relationship between Bending and Cantilever Length

1. Now, measure and record the amount of bending that takes place for at least 8 different lengths of the cantilever. Again determine within your group which lengths you can use to be able to see a different in the amount of bending.

- a. Make a table that lists the amount of force for the eight different amounts of force.
- b. Draw a scatterplot of these values.
- c. By the shape of the plotted points, does this function seem to be linear, quadratic, or neither?

2. Using the Linear Regression, Quadratic Regression, Cubic Regression, and Quartic Regression functions on your calculator, determine which model fits the data the best. Which model seems to fit the data most closely?

- a. Sketch this function over your scatterplot.
- b. In what ways does this function fit your data? In what ways does it not?

3. Write the equation for the model that best fits your data from your calculator. What type of function is this?

- a. Give the domain and range for this function.
- b. Using this function, predict how much the cantilever will bend for ____.
- c. Using this function, predict how much the cantilever will bend if the length of the cantilever was 50 inches.
- d. Does this value make sense? Why or why not?
- e. Recall the domain and range you specified for this function in (f). Consider your answers in parts (b), (g) and (h). Given this problem, how does the context restrict your domain and/or range?

4. Is this function increasing or decreasing? In the context, what does that mean?

5. What type of function is this?

POLYNOMIAL PATTERNS Learning Task:

1. In the Cantilever Learning Task we learned about a third degree polynomial function, also called a cubic function. What do you think polynomial means?

Let's break down the word: poly- and -nomial. What does "poly" mean?

a. A *monomial* is a numeral, variable, or the product of a numeral and one or more variables. For example: -1, $\frac{1}{2}$, 3x, 2xy. Give a few examples of other monomials:

b. What is a *constant*? Give a few examples:

c. A *coefficient* is the numerical factor of a monomial or the _____ in front of the variable in a monomial.

Give some examples of monomials and their coefficients.

d. The *degree of a monomial* is the sum of the exponents of its variables.

has degree 4. Why?

What is the degree of the monomial 3? Why?

e. Do you know what a **polynomial** is now?

A *polynomial function* is defined as a function, $f(x) =$

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-2}x^2 + a_{n-3}x^1 + a_n$, where the coefficients are real numbers.

The *degree of a polynomial* (n) is equal to the greatest exponent of its variable.

The *coefficient* of the variable with the greatest exponent (a_0) is called the leading coefficient.

For example, $f(x) = 4x^3 - 5x^2 + x - 8$ is a third degree polynomial with a leading coefficient of 4.

2. Previously, you have learned about linear functions, which are first degree polynomial functions, $y = a_0x^1 + a_1$, where a_0 is the slope of the line and a_1 is the intercept (Recall: $y = mx + b$; here m is replaced by a_0 and b is replaced by a_1 .)

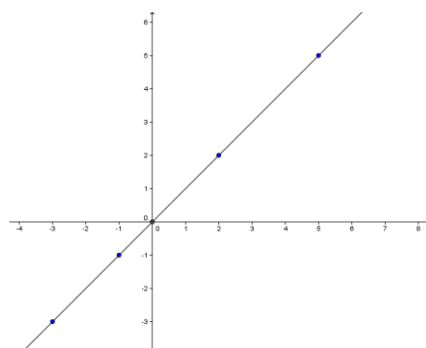
Also, you have learned about quadratic functions, which are 2nd degree polynomial functions, which can be expressed as $y = a_0x^2 + a_1x^1 + a_2$.

In the Cantilever Learning Task you found that a cubic function modeled the data in one of the investigations. A cubic function is a third degree polynomial function. There are also fourth, fifth, sixth, etc. degree polynomial functions.

a. To get an idea of what these functions look like, we can graph the first through fifth degree polynomials with leading coefficients of 1. For each polynomial function, make a table of 6 points and then plot them so that you can determine the shape of the graph. Choose points that are both positive and negative so that you can get a good idea of the shape of the graph. Also, include the x intercept as one of your points.

For example, for the first order polynomial function: $y = x^1$. You may have the following table and graph:

x	y
-3	-3
-1	-1
0	0
2	2
5	5
10	10



b. Compare your five graphs. By looking at the graphs, describe in your own words how $y = x^2$ is different from $y = x^4$. Also, how is $y = x^3$ different from $y = x^5$?

c. Note any other observations you make when you compare these graphs.

3. In this unit, we are going to discover different characteristics of polynomial functions by looking at patterns in their behavior. Polynomials can be classified by the **number** of monomials, or *terms*, as well as by the **degree of the polynomial**. The degree of the polynomial is the same as the term with the highest degree. Complete the following chart. Make up your own expression for the last row.

Example	Degree	Name	No. of terms	Name
2		Constant		Monomial
$2x^2 + 3$		Quadratic		Binomial
$-x^3$		Cubic		
$x^4 + 3x^2$		Quartic		
$3x^5 - 4x + 2$		Quintic		Trinomial

The characteristics of the polynomial functions that you have explored through the Cantilever Learning Task are some important characteristics in being able to describe and to sketch graphs of other polynomial functions. Some of these characteristics we have already examined with linear and quadratic functions, and others will be new.

4. In order to examine their characteristics in detail so that we can find the patterns that arise in the behavior of polynomial functions, we can study some examples of polynomial functions and their graphs. Here are 8 polynomial functions and their accompanying graphs that we will use to refer back to throughout the task. (To be given to you by your teacher)

Each of these equations can be re-expressed as a product of linear factors by factoring the equations.

a. List the x -intercepts of $j(x)$ using the graph above. How are these intercepts related to the linear factors in gray?

b. Why might it be useful to know the linear factors of a polynomial function?

c. Although we will not factor higher order polynomial functions in this unit, you have factored quadratic functions in Math II. For review, factor the following second degree polynomials, or quadratics.

a) $y = x^2 - x - 12$

b) $y = x^2 + 5x - 6$

c) $y = 2x^2 - 6x - 10$

d. Using these factors, find the roots of these three equations.

e. Sketch a graph of the three quadratic equations above without using your calculator and then use your calculator to check your graphs.

f. Although you will not need to be able to find all of the roots of higher order polynomials until a later unit, using what you already know, you can factor some polynomial equations and find their roots in a similar way.

Try this one: $y = x^5 + x^4 - 2x^3$.

What are the roots of this fifth order polynomial function?

g. How many roots are there?

Why are there not five roots since this is a fifth degree polynomial?

h. Check the roots by generating a graph of this equation using your calculator.

i. For other polynomial functions, we will not be able to draw upon our knowledge of factoring quadratic functions to find zeroes. For example, you may not be able to factor

$$y = x^3 + 8x^2 + 5x - 14$$

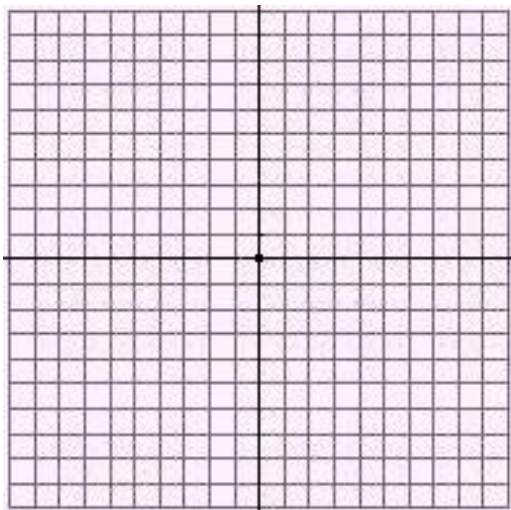
, but can you still find its zeros by graphing it in your calculator?
How?

Write are the zeros of this polynomial function.

5. Symmetry

The first characteristic of these 8 polynomial functions we will consider is symmetry.

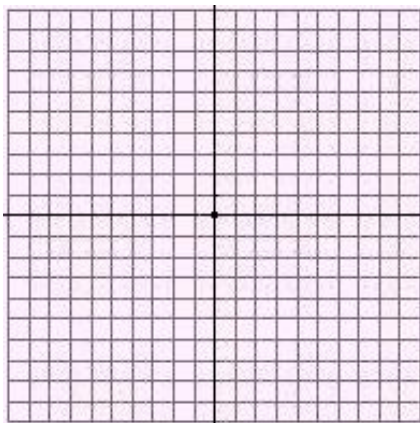
a. Sketch a function you have seen before that has symmetry about the y-axis.



Describe in your own words what it means to have symmetry about the y-axis.

What do we call a function that has symmetry about the y-axis?

b. Sketch a function you have seen before that has symmetry about the origin.



Describe in your own words what it means to have symmetry about the origin.

What do we call a function that has symmetry about the origin?

c. Using the table below and your handout of the following eight polynomial functions, classify the functions by their symmetry.

Function	Symmetry about the y axis?	Symmetry about the origin?	Even, Odd, or Neither?
$f(x) = x^2 + 2x$			
$g(x) = -2x^2 + x$			
$h(x) = x^3 - x$			
$j(x) = -x^3 + 2x^2 + 3x$			
$k(x) = x^4 - 5x^2 + 4$			
$l(x) = -(x^4 - 5x^2 + 4)$			
$m(x) = \frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$			
$n(x) = -\frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$			

d. Now, sketch your own higher order polynomial function (an equation is not needed) with symmetry about the y-axis.

e. Now, sketch your own higher order polynomial function with symmetry about the origin.

f. Using these examples from the handout and the graphs of the first through fifth degree polynomials you made, why do you think an odd function may be called an odd function? Why are even functions called even functions?

g. Why don't we talk about functions that have symmetry about the x-axis? Sketch a graph that has symmetry about the x-axis. What do you notice?

6. Domain and Range

Another characteristic of functions that you have studied is domain and range. For each polynomial function, determine the domain and range.

Function	Domain	Range
$f(x) = x^2 + 3$		
$g(x) = -2x^2$		
$h(x) = x^3 - x$		
$j(x) = -x^3 + 2x^2 + 3x$		
$k(x) = x^4 - 5x^2 + 4$		
$l(x) = -(x^4 - 5x^2 + 4)$		
$m(x) = \frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$		
$n(x) = -\frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$		

7. Zeros

a. We can also describe the functions by determining some points on the functions. We can find the x-intercepts for each function as we discussed before. Under the column labeled “x-intercepts” write the ordered pairs (x,y) of each intercept and record the number of intercepts in the next column. Also record the degree of the polynomial.

Function	Degree	X-intercepts	Zeros	Number of Zeros
$f(x) = x^2 + 3$				
$g(x) = -2x^2$				
$h(x) = x^3 - x$				
$j(x) = -x^3 + 2x^2 + 3x$				
$k(x) = x^4 - 5x^2 + 4$				
$l(x) = -(x^4 - 5x^2 + 4)$				
$m(x) = \frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$				
$n(x) = -\frac{1}{2}(x^5 + 4x^4 - 7x^3 - 22x^2 + 24x)$				

b. These x-intercepts are called the *zeros* of the polynomial functions. Why do you think they have this name?

c. Fill in the column labeled “Zeroes” by writing the zeroes that correspond to the x-intercepts of each polynomial function, and also record the number of zeroes each function has.

d. Make a conjecture about the relationship of degree of the polynomial and number of zeroes.

e. Test your conjecture by graphing the following polynomial functions using your calculator:
 $y = x^2$, $y = x^2(x - 1)(x + 4)$, $y = x(x - 1)^2$.

Function	Degree	X-Intercepts	Zeroes	Number of Zeroes
$y = x^2$		(0,0)		
$y = x^2(x - 1)(x + 4)$		(0,0); (0,-1);(0-4)		
$y = x(x - 1)^2$				

How are these functions different from the functions in the table?

Now amend your conjecture about the relationship of the degree of the polynomial and the number of x-intercepts.

Make a conjecture for the maximum number of x-intercepts the following polynomial function will have: $p(x) = 2x^{11} + 4x^6 - 3x^2$

8. End Behavior

In determining the range of the polynomial functions, you had to consider the *end behavior* of the functions, that is the value of $f(x)$ as x approaches infinity and negative infinity.

Polynomials exhibit patterns of end behavior that are helpful in sketching polynomial functions.

- a. Graph the following on your calculator. Make a rough sketch next to each one and answer the following:
- Is the **degree** even or odd?
 - Is the leading coefficient, the coefficient on the term of highest degree, positive or negative?
 - Does the graph rise or fall on the left? On the right?

1. $y = x$

2. $y = x^2$

3. $y = -3x$

4. $y = 5x^4$

5. $y = x^3$

6. $y = 2x^5$

7. $y = -x^2$

8. $y = -3x^4$

9. $y = -x^3$

10. $y = -2x^5$

11. $y = -3x^6$

12. $y = 7x^3$

- b. Write a conjecture about the **end behavior**, whether it rises or falls at the ends, of a function of the form $f(x) = ax^n$ for each pair of conditions below. Then test your conjectures on some of the 8 polynomial functions graphed on your handout.

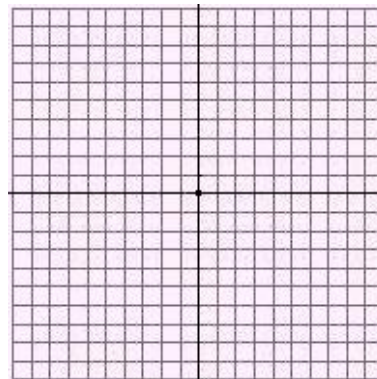
Condition a: When n is even and $a > 0$,

Condition b: When n is even and $a < 0$,

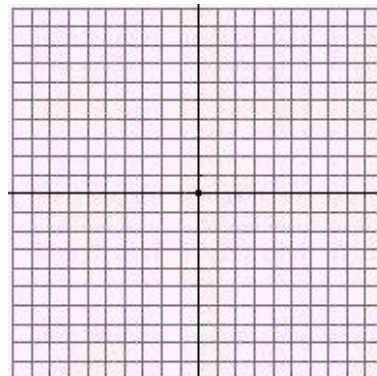
Condition c: When n is odd and $a > 0$,

Condition d: When n is odd and $a < 0$,

- c. Based on your conjectures in part (b), sketch a fourth degree polynomial function with a negative leading coefficient.



- d. Now sketch a fifth degree polynomial with a positive leading coefficient.



Note we can sketch the graph with the end behavior even though we cannot determine where and how the graph behaves otherwise without an equation or without the zeros.

- e. If we are given the real zeros of a polynomial function, we can combine what we know about end behavior to make a rough sketch of the function.

Sketch the graph of the following functions using what you know about end behavior and zeros:

a) $f(x) = (x - 2)(x - 3)$

b) $f(x) = -x(x - 1)(x + 5)(x - 7)$

- 9. Critical Points** Other points of interest in sketching the graph of a polynomial function are the points where the graph begins or ends increasing or decreasing.

Recall what it means for a point of a function to be an *absolute minimum* or an *absolute maximum*.

Which of the twelve graphs from part 8(a) have an absolute maximum?

Which have an absolute minimum?

What do you notice about the degree of these functions?

Can you ever have an absolute maximum AND an absolute minimum in the same function? If so, sketch a graph with both. If not, why not?

For each of the following graphs from the handout, locate the turning points and the related intervals of increase and decrease, as you have determined previously for linear and quadratic polynomial functions.

Then record which turning points are *relative minimum* (the lowest point on a given portion of the graph) and *relative maximum* (the highest point on a given portion of the graph) values.

Function	Degree	Turning Points	Intervals of Increase	Intervals of Decrease	Relative Minimum	Relative Maximum
$f(x)$						
$h(x)$						
$k(x)$						
$n(x)$						

b. Make a conjecture about the relationship of the degree of the polynomial and the number of turning points that the polynomial has. Recall that this is the maximum number of turning points a polynomial of this degree can have because these graphs are examples in which all zeros have a multiplicity of one.

c. Sometimes points that are relative minimums or maximums are also absolute minimums or absolute maximum. Are any of the relative extrema in your table also absolute extrema?

10. Putting it all Together

Now that you have explored various characteristics of polynomial functions, you will be able to describe and sketch graphs of polynomial functions when you are given their equations.

If I give you the function: $f(x) = (x-3)(x+1)^2$, then what can you tell me about the graph of this function? Make a sketch of the graph of this function, describe its end behavior, and locate its critical point and zeroes.

WEIGHT LOSS PROGRAM EVALUATION LEARNING TASK:
Weight Loss Program (also MM3D3)

An organization has established a 12-week weight-loss program that they claim their participants on average lose 24 pounds in those 12 weeks! Here is an advertisement for the program:



Lose Weight Fast

With our 12-week weight loss program, on average participants lose 24 pounds!

Below are our topics for discussion and focus for the program meetings:

Week 1: Healthy weight starts with a healthy diet.

Week 3: Supplementing a healthy diet with regular exercise.

Week 6: Sharing experiences with the group.

Week 9: Motivational guest speaker.

Week 12: Final weigh in and celebration!

SIGN UP TODAY BY CALLING: 123-456-7890

1. Evaluation Questions

You are part of a team that has been asked to evaluate this program and offer information about the strengths and weaknesses of the program. Many times the role of a program evaluator is to display data in meaningful ways to explain it to those who are interested in the results of the evaluation. However, first program evaluators must know what kinds of data to collect in order to answer any questions they may have. In order to know what data to collect, program evaluators need to understand what questions they are trying to answer about the program.

a. What questions might you want answered if you were considering whether or not to join this program?

What are some questions you might want answered if you were the director of this program?

Are there other evaluation questions to consider?

b. There may be many questions considered by the program evaluation team, which may consist of many members. What types of data might you collect to answer each of these questions?

2. Data

Your participation on the team is to analyze the following data set that was collected by a team of researchers. The researchers recorded the data from the weigh-ins that participants do at each meeting *and* chose a class of participants to follow-up on how well they maintained the weight loss after the program by also conducting weigh-ins at three week intervals for 9 weeks once the program was completed. The following table contains the data that was collected on the last class of 5 participants, and they have asked you to summarize and analyze the trends of the data. In order to protect the participants' identities, your team has labeled the participants A through E. The data points in the column labeled Week 0 are the original weights of the participants at the very first meeting.

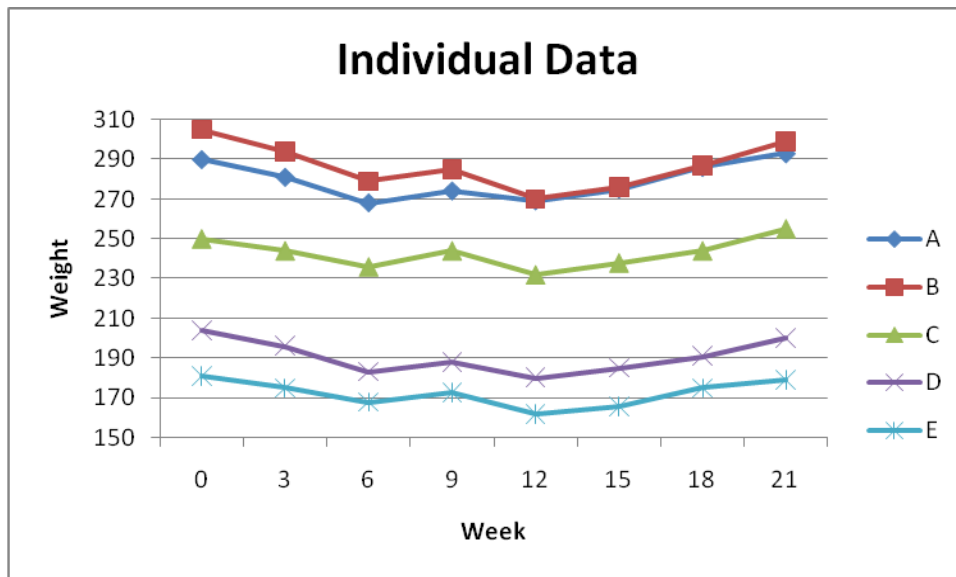
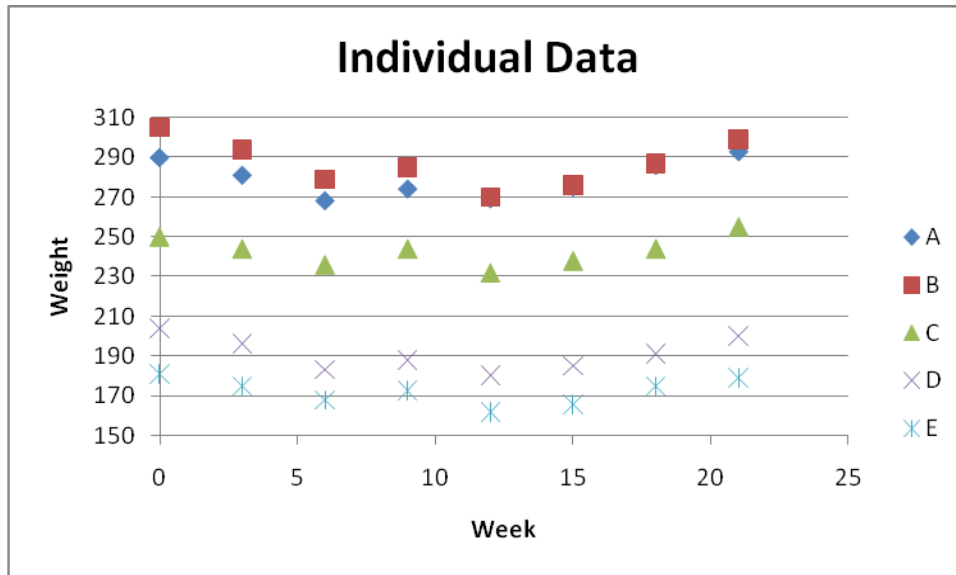
Participants/Week	0	3	6	9	12	15	18	21
A	290	281	268	274	269	275	286	293
B	305	294	279	285	270	276	287	299
C	250	244	236	244	232	238	244	255
D*	204	196	183	188	180	185	191	200
E*	181	175	168	173	162	166	175	179

* Denotes a female participant.

- Which of your above evaluation questions do you think you may be able to answer with this data?
- In what ways do you think it would be useful to represent the above data to examine the nature of the weight loss for the program's participants?

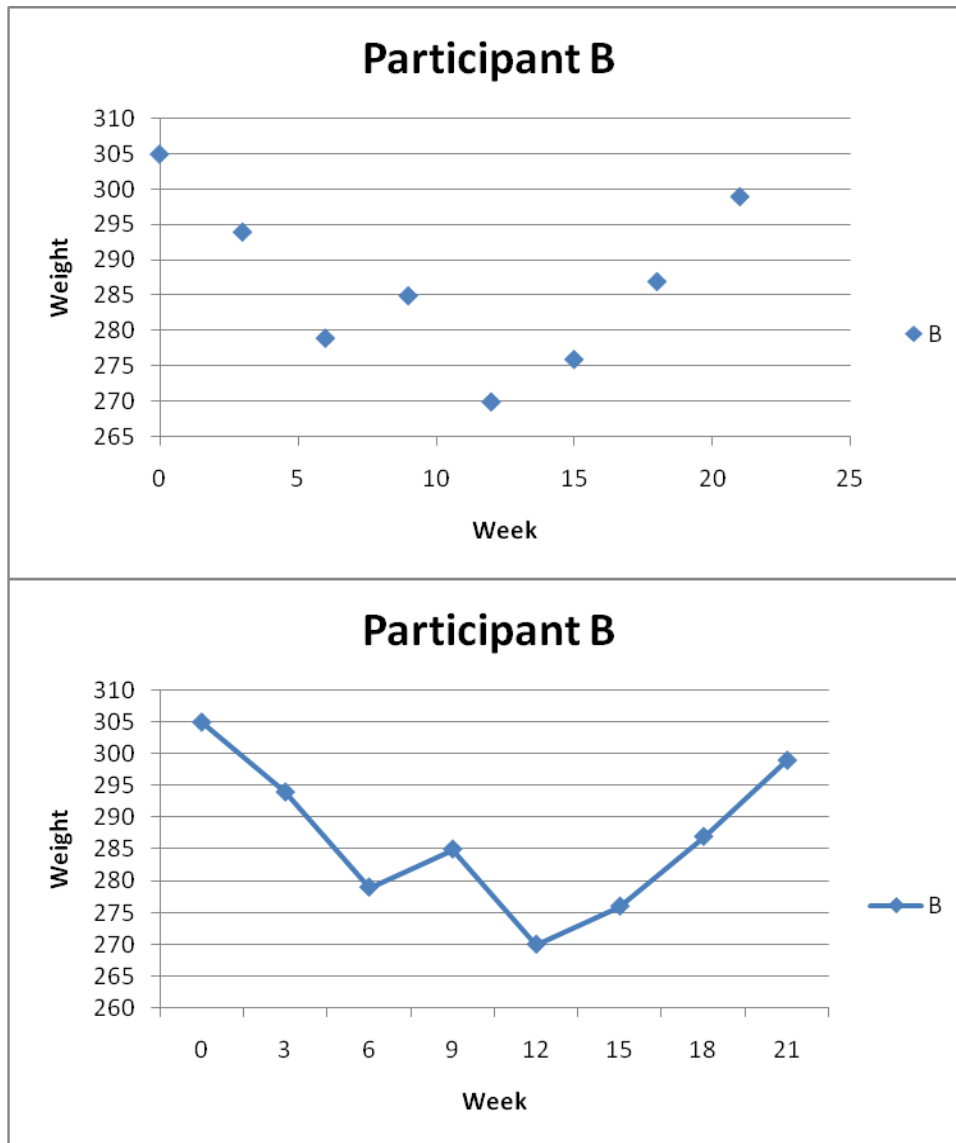
3. Individual Data

a. How did you decide to display the individual data? Why? If you didn't display the individual data, how would you do so?



b. How is this data useful? What trends do you see in the data?

Consider just participant B.



c . During which intervals did Participant B lose weight?

Given the context, why do you think Participant B lost weight during these intervals? (Refer to the schedule of the Weight Loss Program for some ideas.)

d. During which intervals did Participant B gain weight?

Given the context why do you think Participant B gained weight during these intervals?

e. Without fitting a curve to this data and just by looking at the data, what type of function do you think would fit this data?

Why would a linear function not model this data?

Would a quadratic function appropriately model this data?

f. If a polynomial function were to be fit to this data, what degree do you think that function would have? Would the leading coefficient be positive or negative? Why?

Sketch in the polynomial function you would use to model this data over your existing graph or plot.

Can you interpret data points outside of the Week 0 through Week 21 with this function? Why or why not?

g. Does makes sense to use the entire domain and range of this function to model this data? Why or why not?

Given our data, what would the domain and range of the function be that could model this data?

What would be a reasonable range for a similar function that models any adult participant's weight?

h. Given the context, about how much do you think Participant B will weigh 30 weeks after the start of the program?

One year after the start of the program? Why do you think this?

How do these thoughts support restricting the domain of the polynomial function as you did in part (g)?

i. How much do you think Participant B may have weighed at Week 10? Why? Think about the context of the problem when you answer this question.

j. Using set notation, state the intervals of increase and decrease of the function that you drew.

What point is the absolute minimum within for the function you drew to model this data for Participant B? What does this point represent, given the context of the problem?

What point on the scatterplot represents the most Participant B weighed in the time interval data was collected?

What is another point that is a relative minimum for your function? Interpret this point in the context.

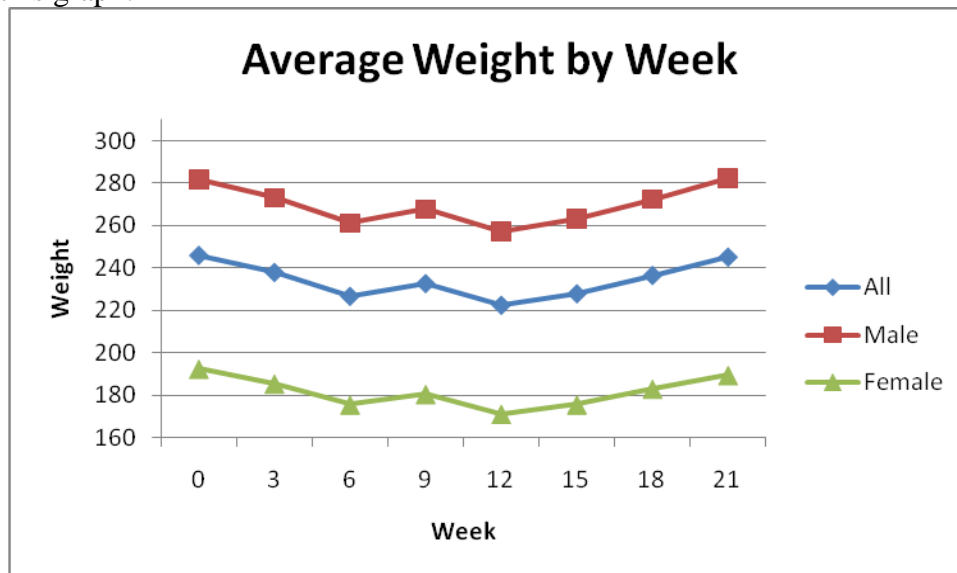
What is one point that is a relative maximum for this function? Interpret this point in the context.

4. Group Data

How did you compile the data to graphically display the data for all the participants with one graph?

How does the Average Female Weight by Week graph differ from the Average Male Weight by Week graph?

It may be helpful to compare the functions that could be used to model these data by looking at this graph:



What information do we gain by separating the averages by male and female?

Sketch in the portion of the polynomial function that can be used to model the male average weight by week data and a separate function to model the female average weight by week data.

In what ways are the functions similar or different in terms of their

Domain and Range:

Intervals of increase and decrease:

Relative minimums and relative maximums:

Suppose you could model the average female weight by week by a polynomial function, $f(x)$. How could you translate this function into a function $m(x)$ that would roughly model the average male weight by week?

Represent this translation symbolically by expressing the function $m(x)$ in terms of $f(x)$. It may help to recall how you would translate the function $g(x) = x$ or $h(x) = x^2$, and then generalize this to this situation with a higher order polynomial function.

Let $a(x)$ represent the function that would model the average weight by week for all participants. Express $a(x)$ as a translation of $f(x)$.

Express $a(x)$ as a translation of $m(x)$.

Do either of these functions have any real zeros? How do you know?

Interpret what a real zero would mean in this context.

5. Report

Given these explorations of the weight program data, compile a report summarizing your findings for the individual, average, and/or net weight loss/data (or any other data you considered) and suggest some strengths and weaknesses of the program. Also include any other data that would have helped you make evaluations about the program that should be included the next time the program is evaluated.

THE SWIMMING DEBATE LEARNING TASK:

1. The Task

Alexis, Loren and Roxy are trying to determine who the fastest swimmer is by timing each others' swims from one end of the pool to the other. The pool is an Olympic size pool with a 50 meter length, 25 meter width, and 2 meter depth as indicated in the drawing below.

Loren swims first; he swims from point S (the starting point will be the exact center of the side of the pool) to point L (the top of the corner of the pool) in 51.538 seconds.

Then Roxy swims from point S to point R (the bottom corner of the pool) in 51.578 second.

Finally, Alexis swims from point S to point A (the top center of the other side of the pool) in 50.039 seconds.

When Alexis hears her time, she exclaims, "Yes, I win! I am the fastest swimmer." Loren then figures, "Well, you may have had the quickest time, but I am the fastest swimmer because I swam farther than you and had a quicker time than Roxy. Roxy rebuts, "Well if that's the case, I swam even farther than Loren did and had almost the exact same time, so maybe I'm the fastest swimmer."

Assuming all measurements were exact and the swimmers swam in a straight line from their starting to finishing points, your task is to help Loren, Alexis and Roxy determine whose rate was the fastest.

Make a sketch of the problem situation.

How can you determine the distance that each person swam?

2. Distance

Recall before having found the distance in two-dimensional space, for example finding the distance that someone has run on a flat surface like a track. This type of problem only has two dimensions, which can be represented by the two dimensional Cartesian coordinate system with 2 axes, which are commonly labeled the x and y axis.

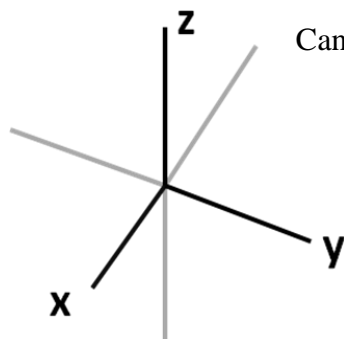
a. How many pieces are there in the 2-D coordinate system?

What are these "pieces" called?

b. For plotting points in 3-D, we will have a Cartesian system with three perpendicular axes: x, y, and z. With these axes we can model the 3-D world in which we live.

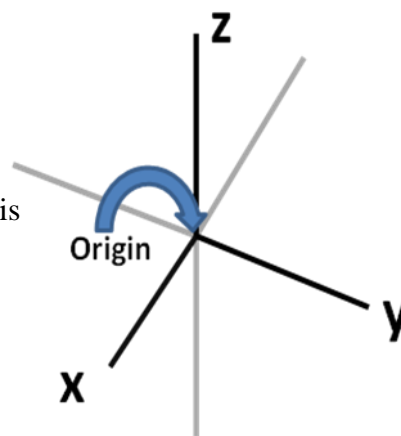
How many "pieces" do you think there will be in the 3-D coordinate system?

To picture the axes, consider a lower corner of your classroom: two walls meet together at a right angle. Consider where the two walls meet to be a piece of z axis, which will represent the height of the room. Where the two walls meet the floor, we can consider as pieces of two axes: the x and the y axes. One represents length, the other width. In the drawing below, you can see that the corner of the room is like the black portion of the axes. The gray portions of the axes represent negative values of the axes.

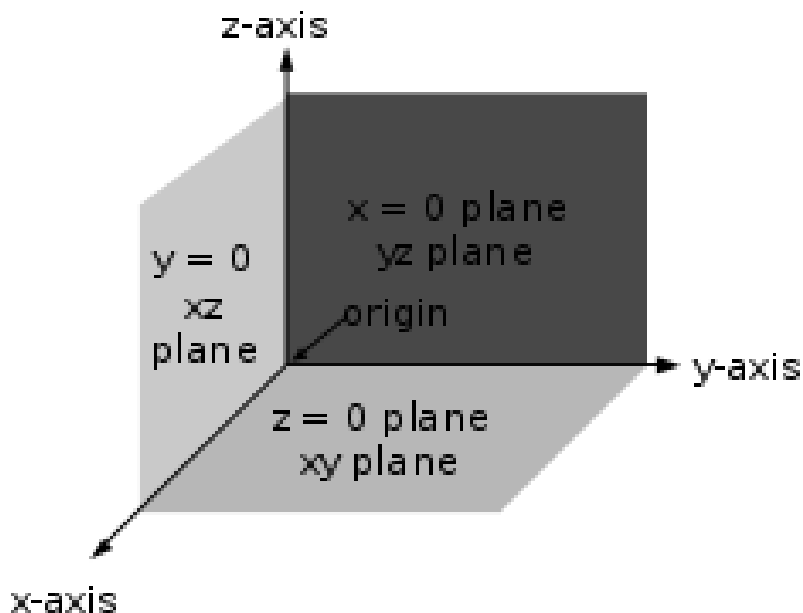


Can you see how many pieces are in the 3-D coordinate system now?

Your classroom is in these pieces called octants. We can designate that this octant has all positive x, y, and z values. For this representation, we can think of the lower corner of the room as the origin of the three dimensional coordinate system, where all three axes intersect. The origin is the point in space with coordinates $(0, 0, 0)$.



The two walls and the floor each individually lie on a two dimensional plane. Consider the floor first: If there is no height, then the z dimension equals 0 and we are left with only length and width, which is the plane that the floor lies on (the x-y plane). If you have no length along the y-axis, then we are left with the x-z plane as pictured below. Similarly, without width along the x-axis, we have the y-z plane.



To describe a point in 3-D space, we use ordered triples, (x, y, z) . For example, the ordered triple of $(2, 7, 8)$ describes a unique point in space. You can find this point by going 2 units in the positive direction along the x-axis, then 7 units in the positive direction along the y axis, and finally 8 units in the positive direction along the z axis.

c. Group Activity

Use the corner of your classroom to represent the origin of 3-D space. Place a cardboard box into the corner of your classroom. Your group's goal is to determine the ordered triples that describe the locations of the vertices of the box in space.

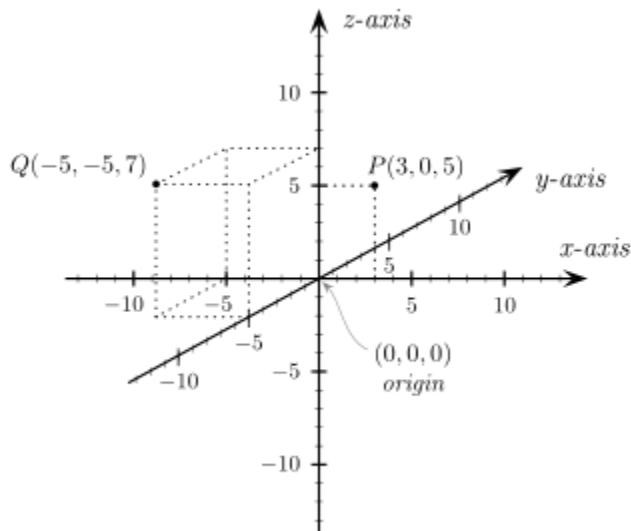
1. By measuring its sides, determine the dimensions of the box. Record the dimensions on paper.
2. How many vertices does the box have? Label each vertex and make a sketch of the box on paper.
3. Using inches as your units, find the 8 ordered triples that describe each of box's vertices.
4. Record these ordered triples on a separate piece of paper. Trade your paper for another group's paper that has the ordered triples that describe the size of their box. Using only their coordinates, determine the dimensions of their box.
5. Make a sketch of the other group's box and include the lengths of each side.

6. Check back with the group to see if the dimensions you came up with match the dimensions they measured.

7. This figure shows a box which is situated at the origin.
Determine the coordinates of vertices A, B, and C
using the dimension of the box, which are labeled.

8. Describe in your own words how you can find the coordinates of any point in space. Does thinking about that point as the corner of a box that is placed with one corner at the origin help you visualize the problem? How so?

Although we have not yet dealt with negative values of x , y , or z thus far, points are plotted similarly with positive and negative coordinates, as demonstrated below. This figure demonstrates two points, $P(3, 0, 5)$ and $Q(-5, -5, 7)$ plotted in three dimensional space.



b. Sketch a plot of the following ordered triples:

- $(4, 2, 7)$
- $(-8, 0, 3)$
- $(-3, -1, -4)$
- $(5, -3, 2)$

- e) Make up your own coordinates and try plotting the point.

Now that you know how to plot points in three dimensional space, we are one step further to being able to calculate a distance in space, which is required by our problem we are trying to solve. Let's return to the sketch a graph of our problem situation.

Let the starting point, S, be the origin of the coordinate system.

Describe the points L, A and R with an ordered triple by letting the x-coordinate be the length, the y-coordinate be the width and the z-coordinate be the depth.

What are the three distances that the problem requires us to calculate? Write the line segments that represent these distances from your drawing.

How will calculating these three distances be different from each other?

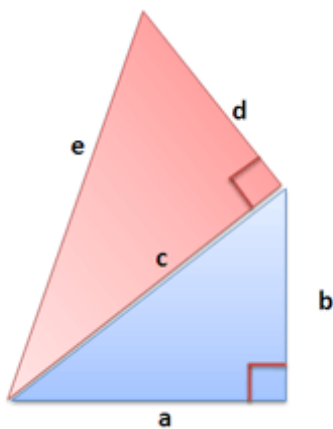
Can you already find any of these distances with what you already know? If so, go ahead and calculate them now.

In order to determine a 3-dimensional distance, we can review how we calculated distance in two dimensions and extend that idea to three dimensions.

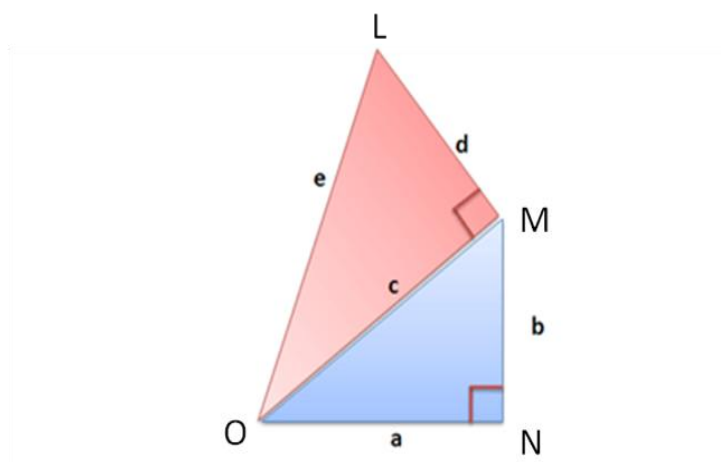
Recall how to calculate distance in two dimensions. Determine the distance from A (-3,-1) to B (2,5). Using a graph, show how you know your answer is correct. Include the name of the Theorem that we use to show this.

Now that we recall the derivation of the distance formula, we can extend that idea for three dimensions.

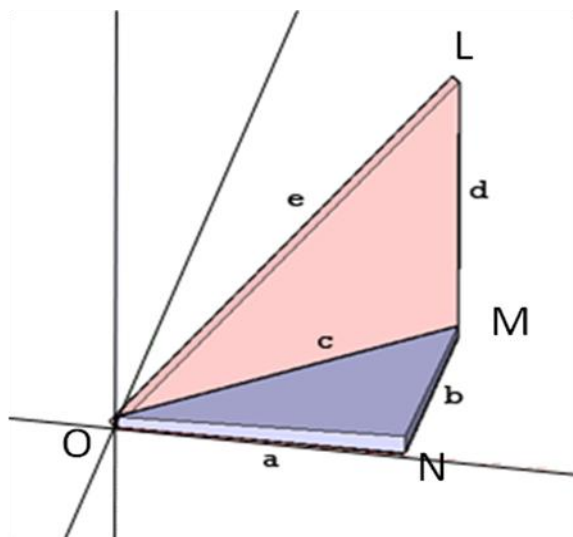
- f. Write the equation for how you would find c^2 in the figure below. Write a similar equation for e^2 . Then, write an equation expressing e in terms of a , b , and d .



If we label the quadrilateral that the two triangles make up, as shown below, we have found the length of segment LO by using the Pythagorean theorem.



Now, imagine one of the triangles coming out of the paper. Tilt triangle OML up:



It's the same triangle, just facing a different way. But now we're in 3D! The distance formula you found for finding \overline{LO} is the same distance formula you would use to find the distance from point L to point O in this 3-D picture:, where a , b , c , d , and e are the lengths sides of the right triangles as shown above.

If the coordinates of points O, N, M and L are (o_1, o_2, o_3) , (n_1, n_2, n_3) , (m_1, m_2, m_3) , and (l_1, l_2, l_3) , respectively, then we can re-express this distance formula $a^2 + b^2 + d^2 = e^2$ in terms of the coordinates of the points instead of the length of the sides so that we can see exactly how and why this formula works:

The length of segment \overline{ON} which is labeled a represents a length along the x-dimension, so we can express this length in terms of the x-coordinates of points O and N:

The length of segment \overline{NM} which is labeled b represents a length along the y-dimension, so we can express this length in terms of the y-coordinates of points N and M:

The length of segment \overline{ML} which is labeled d now represents a length along the z-axis, so we can express this length in terms of the z-coordinates of points M and L:

The length of segment \overline{LO} which is labeled e is the distance we are trying to find.

Now using this information, re-express $a^2 + b^2 + d^2 = e^2$ in terms of the coordinates of points O, L, M, and N:

We are now able to find the distance between two points, if we know their coordinates, by visualizing the two triangles and using the Pythagorean Theorem.

Now, you can apply what you have learned about finding distance in a three dimensional space to help Loren, Alexis, and Roxy determine who swam at the fastest rate.

Calculate the distance each swimmer swam. Show how the distance formula was used for each calculation. Write Roxy's distance in terms of the coordinates of the points s, A, L, and R.

$$\overline{sA} = \sqrt{(50 - 0)^2 + (0 - 0)^2 + (0 - 0)^2} = 50 \text{ feet}$$

$$\overline{sL} = \sqrt{(50 - 0)^2 + (12.5 - 0)^2 + (0 - 0)^2} = 51.539 \text{ feet}$$

$$\overline{sR} = \sqrt{(\overline{as})^2 + (\overline{al})^2 + (\overline{lr})^2} =$$

$$\overline{sR} = \sqrt{(a_1 - s_1)^2 + (l_2 - a_2)^2 + (l_3 - r_3)^2}$$

$$\overline{sR} = \sqrt{(50 - 0)^2 + (12.5 - 0)^2 + (2 - 0)^2} = 51.578 \text{ feet}$$

Use this information to determine who swam the fastest. Record the distances, times, and rates in a table.

Who had the fastest time?

Who had the fastest rate?

Who swam the longest distance?

Explain to Alexis, Loren, and Roxy who was the fastest swimmer and why.

Building a Swimming Pool

Alexis wants to have a pool in her backyard so that she can practice swimming at her house. Even though she knows she doesn't need one as big as an Olympic size swimming pool, she does want to have all of the dimensions proportional to the dimensions of an Olympic size swimming pool.

If the length of her pool is going to be x , what will be the width of her pool?

What will be the depth of her pool?

Express the volume Alexis's pool with a length of x meters

Make a sketch of this function.

On what intervals is the function increasing? Decreasing?

How would you describe the end behavior of this function? Is this what you would expect given the degree and lead coefficient of this function?

What are the zeroes of this function? Give the multiplicity of each zero.

Explain in your own words how you determined the multiplicity of the zeroes you found.

If you build a pool that is 30 meters long, what is the volume of the pool? Plot this point on your graph and label it P.

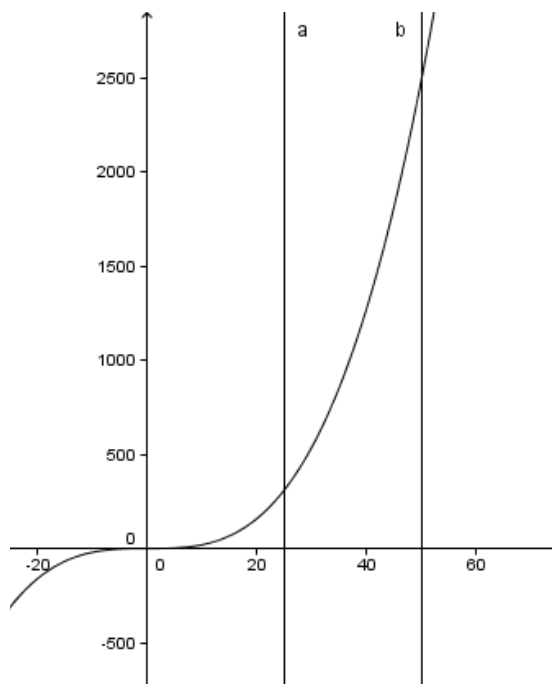
If you build a pool that is 1.5 meters deep, what is the volume of the pool? Plot this point on your graph and label it Q.

Consider how shallow a pool can be and still be able to swim in it. What should the lower bound on the depth of the pool be?

Put a restriction on x , the length of the pool, due to this lower bound on the depth. What would the lower bound for the length be? Write this in terms of x .

What is the upper bound on the length?

Now, graph these two restrictions with your polynomial function for volume. What is your restricted domain and range?



Using this graph, determine one possible set of dimensions that the new pool may have.

Plot the point on the graph that corresponds to these dimensions. Call the point P and label the x and y values.

Shade the area that contains all possible values that x can be.

Alexis's parents decided they would like the pool to hold less than 300,000 gallons of water. Add a constraint to your graph so that the volume equals 300,000 gallons of water. [1 cubic meter = 264.172 gallons].

What is the longest pool Alexis can have with this constraint? Plot this point on your graph.